Binary Star Orbits. III. In which we Revisit the Remarkable Case of Tweedledum and Tweedledee

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ABSTRACT

Two of the most challenging objects for optical interferometry in the middle of the last century were the close components (FIN 332) of the wide visual binary STF2375 (= WDS 18455+0530 = HIP 92027 = ADS 11640). Each component of the wide pair was found to have subcomponents of approximately the same magnitude, position angle and separation and, hence, were designated by the tongue in cheek monikers "Tweedledum and Tweedledee" by the great visual interferometrist William S. Finsen in 1953. They were later included in a list of "Double Stars that Vex the Observer" by W.H. van den Bos (1958a).

While speckle interferometry has reaped a rich harvest investigating the close inteferometric binaries of Finsen, the "Tweedles" have continued to both fascinate and exasperate due to both the great similarity of the close pairs as well as the inherent 180° ambiguity associated with interferometry.

Detailed analysis of all published observations of the system have revealed several errors which are here corrected, allowing for determination of these orbital elements which resolve the quadrant ambiguity. A unique software filter was developed which allowed subarrays from archival ICCD speckle data from 1982 to be re-reduced. Those data, combined with new and unpublished observations obtained in 2001-9 from NOAO 4m telescopes, the Mt. Wilson 100in

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14. ABSTRACT
Two of the most challenging objects for optical interferometry in the middle of the last century were the close components (FIN 332) of the wide visual binary STF2375 (= WDS 18455+0530 = HIP 92027 = ADS 11640). Each component of the wide pair was found to have subcomponents of approximately the same magnitude, position angle and separation and, hence, were designated by the tongue in cheek monikers ?Tweedledum and Tweedledee? by the great visual interferometrist William S. Finsen in 1953. They were later included in a list of ?Double Stars that Vex the Observer? by W.H. van den Bos (1958a). While speckle interferometry has reaped a rich harvest investigating the close inteferometric binaries of Finsen, the ?Tweedles? have continued to both fascinate and exasperate due to both the great similarity of the close pairs as well as the inherent 180 ambiguity associated with interferometry. Detailed analysis of all published observations of the system have revealed several errors which are here corrected, allowing for determination of these or- bital elements which resolve the quadrant ambiguity. A unique software filter was developed which allowed subarrays from archival ICCD speckle data from 1982 to be re-reduced.

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telescope and the Naval Observatory Flagstaff Station 61in telescope as well as high quality unresolved measures all allow for the correct orbits to be determined. Co-planarity of the multiple system is also investigated.

Subject headings: binaries:general—binaries:visual—techniques:interferometry—stars:individual (HR 7048)

1. The discovery and early measures of Tweedledum and Tweedledee

The bright star HR 7048 [= HD 173495 = HIP 92027 = ADS 11640 = STF2375, $(\alpha,\delta) = 18^{\rm h}45^{\rm m}28\overset{\rm s}{.}4$ $+05°30'00''$ (2000)] was first recognized as a double by F.G.W. Struve in 1825 (Struve 1837). Since that time the system has been well observed by many double star astronomers, and has probably been most useful for those wishing to characterize or calibrate their observational systematics, as the motion has long been recognized as quite slow. As early as the start of the last century, Burnham (1906) in his double star catalog (where this object is listed at # 8776) noted “no change in distance, and but little, if any, in the angle.” Almost a century later we have seen a cumulative change of only 12° and $0\overset{''}{.}3$ since the discovery epoch. The first indication that this system might be more than just a slowly moving pair came in a compelling note to Aitken’s (1932) catalog which stated that the “radial velocity of A is variable with a range of 99 km/sec,” citing no less an authority than Plaskett et al. (1921) as the source. The source of the variability — and the system’s interest — was discovered by William Finsen some two decades later.

After experimenting with different interferometer designs, Finsen (1964a) had constructed an eyepiece interferometer, where the observer visually measured interferometric fringe visibilities, then calculated position angles and separations. This instrument, as with other interferometric techniques, was best suited to brighter stars and therefore, a program commenced to investigate the duplicity of all 8,117 stars brighter than magnitude 6.5 with $+20° < \delta < -75°$. In addition to measuring many thousands of known systems, application of this new technique starting in 1951 (Finsen 1951) led to the discovery of 79 new pairs (Mason et al. 2001) almost all of which are close and astrophysically interesting due to their rapid motion.

However, upon turning to the wide components of the Struve pair, Finsen was initially surprised and confused. His first observations were rather vexing, as he reported in his article *A case of Tweedledum and Tweedledee*[1] (Finsen 1953, 1954):

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[1]Tweedledum and Tweedledee are nursery rhyme characters whose names first appeared in an epigram

> "The two pairs are remarkably similar; in fact the simultaneous disappearance of both sets of fringes gave rise to considerable dismay till careful checking showed there was nothing wrong with the instrument."

Apparently, Finsen was quite careful and did not trust his result fully until independently confirmed and measured (albeit crudely) with a micrometer by van den Bos (1956) two nights later. These measures were also quoted in Finsen's discovery paper (1953, 1954). The systems were observed and reported on a fairly regular basis in the 1950s and early 1960s, with eyepiece interferometry by Finsen and with micrometry on large refractors and reflectors by van den Bos, van Biesbroeck, and Muller (see Tables 1 and 2 for all measures and references). Both systems then disappeared from sight (like the Cheshire Cat?) as reported by Finsen (1965, 1967, 1969), van den Bos (1963b), and Worley (1972), although Walker (1969) listed a measure for Ba,Bb (then designated CD) obtained in 1966.

Throughout this time STF2375 retained considerable interest and was among six systems described in some detail by van den Bos (1958a) in his article *Double Stars that Vex the Observer*, where he elaborated a bit upon Finsen's discovery:

> "... When inspecting ADS 11640, Finsen was startled to see the fringes on both components of the Struve pair disappear simultaneously when rotating the interferometer. He suspected, at first, that something had gone wrong with the instrument, but other stars showed nothing abnormal and it turned out that he had indeed found, not fraternal but identical twins, for which he applied the nicknames 'Tweedledum and Tweedledee.'
>
> I have recently measured this object with the Lick 36-inch refractor which clearly separates the two close pairs and the appearance is astonishing. Apart from the fact that Tweedledee ... is slightly fainter than Tweedledum ..., I can see no difference between the two ..."

2. Getting too close to resolve

From the first work of John Herschel (1847), through the large survey of Rossiter (1955) and the work of Finsen and van den Bos at Union and (later) Republic Observatory, double star work at the Cape could be characterized in one of two ways: excellent or inactive. The

by John Byrom (1692–1763). They are best known as a pair of identical twins reciting these rhymes in Lewis Carroll's *Through the Looking Glass and what Alice Found There* (Dodgson 1871).

disappearance of the Tweedles in retrospect seemed to portend a period of benign neglect at the Cape, as van den Bos left for the United States and Finsen approached retirement with some trepidation, as he wrote to Charles Worley (1968a):

> "... There is a move afoot to give first and absolute priority to a programme of planetary photography, to the distress of van den Bos and myself ... and this seems likely to ring the death knell of our long record of intensive double star observing. I found it impossible to explain to people with no experience of double star observing of the demands it places on the observer's skill, enthusiasm and energy to relegate that to a second priority time-filling role is very discouraging, to say the least of it, and may very well kill it stone dead. Time will show."

As Finsen predicted, double star astronomy in South Africa saw a definite downturn after his retirement. Fortunately, some ten years after the demise of eyepiece interferometry the technique of speckle interferometry was developed. In the late 1970s one of the authors (H.A.M.) began a healthy correspondence with W.S. Finsen just as his speckle program was getting started, regarding objects which would be suitable for speckle interferometry. Finsen's continued interest in this pair is apparent in his letter of 1977:

> "... I was reminded of the quadruple that I have dubbed 'Tweedledum and Tweedledee' ... Have you got this on your programme? It would be fun if you could follow it up and eventually do the orbits. These 'identical twins' caused me much agony of mind before I was prepared to accept their duplicity as real. I measured them regularly until 1963 when both became too close to measure without much change in position angles."

3. Speckle Interferometry: The reappearance of the Tweedles

Due to their spatially close nature, many of the systems first resolved by Finsen have orbital periods of less than 50 years. Thus speckle interferometry became a mature technique at an optimal time for orbital analysis of many of Finsen's discoveries; observation of the Finsen stars was therefore given high priority in the early years of this technique. Early results of those efforts include orbital analyses of FIN 342 (McAlister et al. 1988), FIN 312 (Hartkopf et al. 1989), FIN 331, 325, 350, 381 (Hartkopf et al. 1996), FIN 347 (Mason et al. 1996), FIN 359 (Mason 1997), FIN 47 and 328 (Mason et al. 1999).

FIN 332 Aa,Ab and Ba,Bb were both recovered by speckle interferometry in 1976, and continued to be observed on numerous occasions by this and other interferometric techniques (see Tables 1 and 2 and reference quoted therein).

Figure 1 presents a demonstration of the similarity in spatial characteristics of the Tweedles in a "Ferris Wheel" plot. In this diagram the two pairs are shown relative to each other and to the same scale and orientation. The small ellipse in the lower left is the calculated orbit of FIN 332 Aa,Ab while the one in the upper right is Ba,Bb. The large dashed ellipse is an indication of the motion of the wider pair, although given the very small coverage of the orbit it is only present to give an idea of the relative scales of the orbits. The axes are in seconds of arc. The orbits of the close pairs are described in §5.1.

4. Measures of the Tweedles

Tables 1 and 2 present the observations of FIN 332 Aa,Ab and Ba,Bb respectively. Columns one through four contain data specific to the observation: the epoch of observation (expressed as fractional Besselian year), position angle (in degrees), separation (in seconds of arc), and number of measures comprising this mean published position. Note that the position angle has not been corrected for precession and is thus based on the equinox for the epoch of observation. When the pair is unresolved the lower limit on separation is given in column three if published or determined here. Columns five and six give O−C orbit residuals (in θ and ρ) to the orbits presented in §5.1. When the components are unresolved, the O−C columns (five and six) now give, in parentheses, the position predicted by these orbits. The method of observation is indicated in column seven, while the reference to the measure is in column eight. The Center for High Angular Resolution Astronomy (CHARA) photographic speckle camera was less sensitive than the ICCD system, as seen by the small number of measures of the fainter Ba,Bb pair (N=8) vs. Aa,Ab (N=17). Finally, column nine is reserved for the many notes to the measures. In addition to quadrant flips indicated by the correct determination of this previously ambiguous characteristic, there are also other cases where the originally published measures have been corrected. These are described in §4.1.

A representation of the similarity of measurements of these systems to each other is presented in Figures 2a and 2b. Note that the predicted separation and position angle differences (assuming an arbitrary quadrant, i.e., $\pm 180°$) are usually quite small, especially at the time of the discovery and during the first phase of resolutions (§1) where $\Delta\rho < 0''.04$ and $\Delta\theta < 3°$. The two curves and shaded regions, representing the orbital solutions, are presented below.

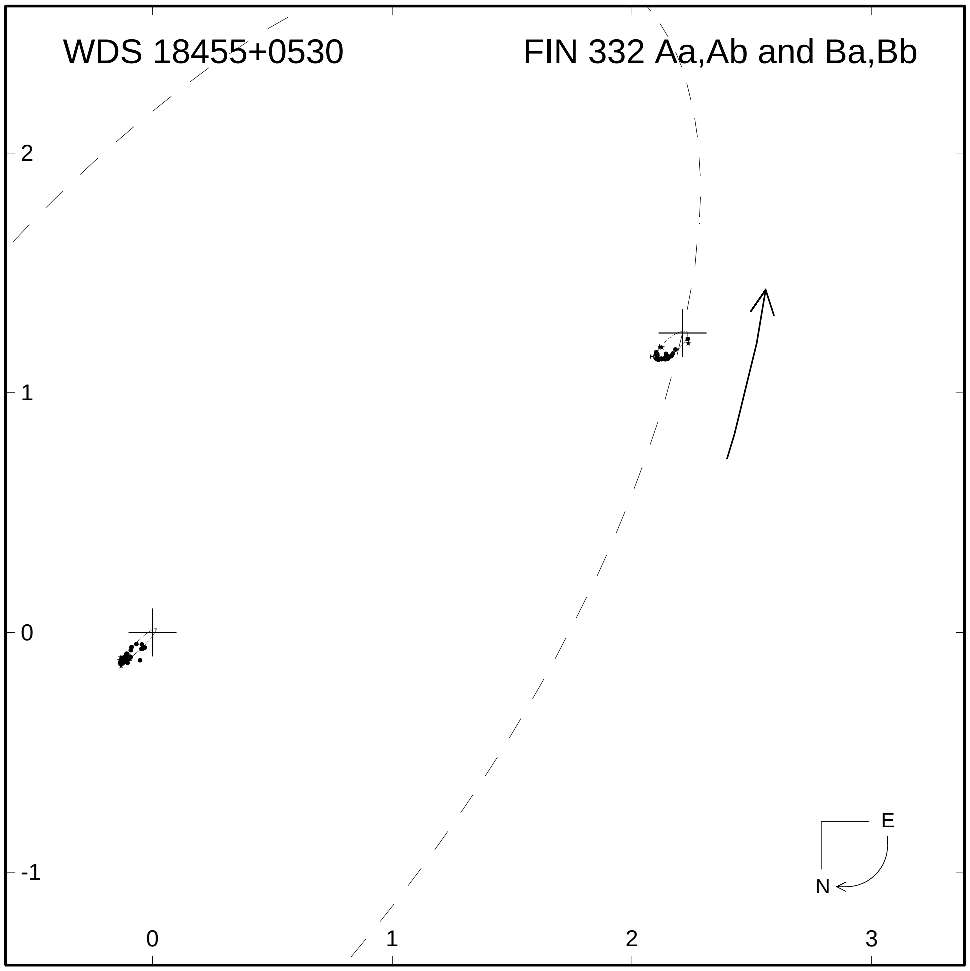

Fig. 1.— A “Ferris Wheel” plot of FIN 332 Aa,Ab and Ba,Bb, shown to the same scale as that of the wide pair, STF2375. These are the orbits of Figures 5 and 6. In this plot the relative positions of STF2375 A and B are fixed and the dashed curve is indicative of the pair’s orbital motion (although it has only moved 12° since its discovery in 1825, so no believeable orbital elements can be determined). The amount and direction of motion of the AB pair over the past 185 years are indicated by the thick curved arrow. The arrow at lower right indicates the direction of motion of both close pairs, which is opposite that of the wide pair. Scales are in arcseconds.

Table 3 provides measures, contemporaneous with those new measures presented here, of the wider AB parent pair, STF2375. Columns one through four are as Tables 1 and 2. Column five provides the method while Column 6 the notes. In this case, the notes simply indicate which telescope was used.

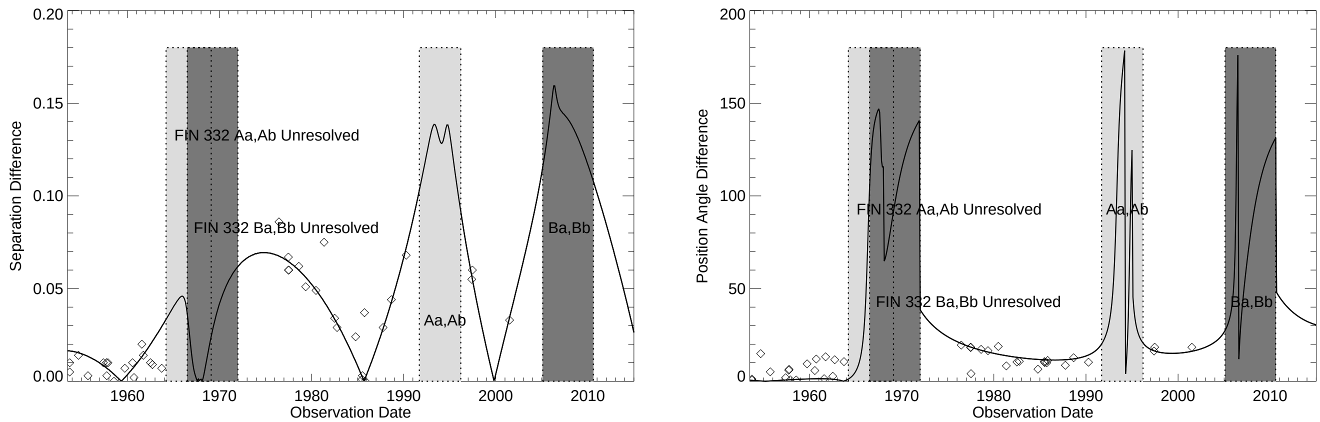

Fig. 2.— The difference in separation and position angle between FIN 332 Aa,Ab and Ba,Bb. The solid curve indicates the predicted difference based on the orbits of §5.1. The diamonds represent the differences between measures of Aa,Ab and Ba,Bb when they were made at the same time by the same observer. The lighter shaded areas from 1964.2–1969.1 and 1991.7–1996.2 are dates when Aa,Ab is predicted to be closer than $0\farcs05$. The darker shaded areas from 1966.5–1972.0 and 2005.1–2010.6 indicates the range of dates when the orbit predicts Ba,Bb to be closer than $0\farcs05$.

4.1. Corrections to Published Measures

A total of seven measures (Aa,Ab: 4, Ba,Bb: 3) were initially published by CHARA using a preliminary calibration. This calibration was corrected in McAlister et al. (1989) and the corrected measures first appeared in McAlister & Hartkopf (1988); the corrected measures are listed here. While the very small Δm of Aa relative to Ab and of Ba relative to Bb presents one set of unique problems, another is the small (but easily measurable) difference between the different components in the wide STF2375 system. Normally, the field-of-view is such that it is possible to observe both pairs; however, problems with analysis of the complex system (see §4.4 below), led the CHARA collaboration to observe this system under high magnification so that only one pair was resolved at a time. In this case, a measure of the Ba,Bb pair (Hartkopf et al. 1997) was incorrectly assigned to Aa,Ab.

Among the most difficult sets of observations to sort out were the 1984 and 1985 observations of the Ba,Bb pair made by Tokovinin & Ismailov (1988). After investigating numerous possible quadrant flips and/or identification errors for these measures and incorporating the unresolved measures in the analysis the two measures did not fit any orbital analysis obeying Kepler's Laws. The first author was consulted, and it is possibly most instructive at this point to quote directly from his response (Tokovinin 2001):

> "I observed it myself at the 1m telescope in 1981, 1984, 1985 with the phase grating interferometer. Both close pairs fell within the focal aperture, so for this object I had to de-center and hide either Aa,Ab or Ba,Bb (in your notation) behind the diaphragm, to get the visibilities of the remaining pair. It is very unlikely that I misidentified the close pairs... For pairs of this separation, the curve of visibility vs. θ has two rather similar maxima. Apparently, in reducing the 1984–85 Ba,Bb data I took the wrong one: this changes the P.A. by roughly 90°, and gives similar, but wrong separation. The choice of the 'correct' maximum was often guided by the previous measurements, and, apparently, in this case was wrong! So, the data on Aa,Ab as measured in 1984–85 must be still valid, and not attributed to Ba,Bb. Measurement error, however, can still be too big, compared to the 4m speckle, because it's a difficult object, it was de-centered, etc. ..."

Given this, it is not surprising, despite the 90° adjustment, that these measures had residuals judged too large by the orbit calculation.

4.2. Hipparcos

The Hipparcos satellite (ESA 1997) observed STF2375 and resolved the wider AB pair and the Ba,Bb pair at the calculated date of 1991.25. The Aa,Ab pair was not resolved. Due to the substantial orbital motion of the Ba,Bb pair during the course of the Hipparcos mission, the quality of this measure may be somewhat degraded.

While all components have the same parallax and proper motions [π = 4.60±1.10 milliarcseconds (mas), μ_α = 15.54±1.07 mas, μ_δ = 1.96±0.86 mas (ESA 1997) π = 5.30±0.85 milliarcseconds (mas), μ_α = 14.32±1.04 mas, μ_δ = 0.16±0.87 mas (van Leeuwen 2007)], the errors are probably larger than this, possibly by as much as 50% (see Urban et al. 2000). While the reasons for the error underestimation may be complex, it is likely that long term motion of wide pairs may not be fully characterized in the few year Hipparcos solution. The Tycho-2 (Høg et al. 2000a,b) proper motion, which is possibly more accurate for long period doubles like AB, is μ_α = 9.4±2.0 mas, μ_δ = −2.5±1.9 mas. Normally a system this bright would have many historical measures to improve the proper motion. However, as the AB system was judged a close pair it was left off many transit circle programs; and only three historical measures, Albany 10, AGK2 and AGK3, were used in the Tycho-2 proper motion determination; all of these were photocenter observations (Urban 2002). Of greater concern is the large Δm assigned by Hipparcos to the Ba,Bb pair of H_p = 0.76 mag. Although the errors are large (Ba = 7.192±0.155 mag, Bb = 7.952±0.315 mag), it is certainly difficult to reconcile this large Δm with the many visual estimates. It may be the result of too many free parameters for four physically related components, even though only three were resolved.

4.3. The Magnitude Difference

Based on published pre-speckle magnitude difference estimates, Aa,Ab has a mean Δm of 0.008$\pm$0.119 while Ba,Bb has a mean of 0.043$\pm$0.232. This quadrant ambiguity can result in two consistent results: one solution is of period P and high eccentricity and, contrariwise (as one of the Tweedles might say), another solution is of period $2P$ and low eccentricity. While we have contemporary measures of Δm (see the notes to Table 1 & 2) which are larger, this only gives the absolute orientation at a single epoch; establishing which orbit is correct requires information from data at either end of the long period solution. The quadrant analysis of Bagnuolo et al. (1992) was successfully used on FIN 342 (McAlister et al. 1988), another binary of small Δm, and this method was utilized here to definitely establish the correct quadrant for both pairs using both historical CHARA ICCD speckle data and more recent United States Naval Observatory (USNO) ICCD speckle data. While preliminary analysis (Mason & Hartkopf 2002) generated long- and short-period solutions for both close pairs, the short-period, high-eccentricity solution has now been determined to be correct in both cases.

4.4. New Old Measures

The first two measures taken with the CHARA CCD system were obtained at a relatively low magnification, such that both of the wider components were observed in the same dataset. As a result of their similar morphologies the closer subcomponents were found to overlap in Fourier space. Of the thirteen peaks ($n(n-1)+1$) expected to be seen in autocorrelation space for a quadruple, only nine were seen. Figure 3a is the measured system geometry at the time of this observation and Figure 3b illustrates the resulting autocorrelogram. Figure 3c is the actual “full frame” directed vector autocorrelation (DVA) of the 1982 data.

In mid-2007, Ellis Holdenried[2] developed software for calculating the DVA of a user-defined subarray of a CCD frame. Review of the archived videotape data, obtained in 1982 and dubbed in 1995, seemed to indicate that the tracking of the telescope was adequate and seeing was good, such that the selected subarray could be static rather than dynamic.

[2]USNO, retired.

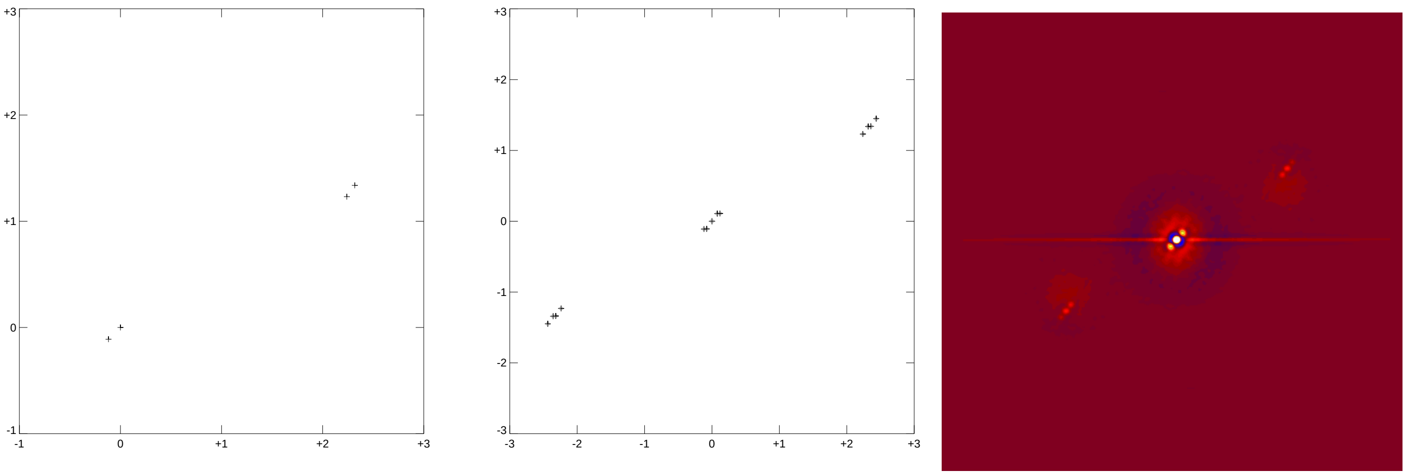

Fig. 3.— Panel a (left) is a representation of the geometry of the system at 1982.7650 based on the new measures of the 1982 data. The scales are in seconds of arc. The origin is the location of the Aa component in relative astrometry space. Panel b (center) is the autocorrelation of a. Note the four visible "double + signs" which represent the blends described in §4.4. The central peak is the zeroth order autocorrelation spike. The gray circles, barely visible at this scale, are $0''.030$ in diameter to indicate regions where detail cannot be seen due to the resolution capabilities of the telescope. Panel c (right) is a digitization of the 1982.7650 data with the blended images. While some of the peaks are quite faint, all nine visible peaks of b are seen here.

While there was some degradation in the video signal, there is significant past experience in working with these old data and recovering good science (Hartkopf et al. 2000). Results of the application of the Holdenried subarray DVA for the two pairs are illustrated in Figure 4.

4.5. New Measures

Additional observations made with the 4m telescopes of Kitt Peak National Observatory and Cerro Tololo Interamerican Observatory were obtained with the USNO speckle camera in 2001 and annually from 2005 to 2008. The system was also observed in 2006–07 with the Mt. Wilson 100in telescope and in 2004 and 2008 with the Naval Observatory Flagstaff Station 61in telescope.

Also re-reduced was an April 1996 observation with the CHARA ICCD of Aa,Ab. The observation has been initially inspected with no measure obtained (Ba,Bb was published in Hartkopf et al. 2000). Reanalysis of the archived videotape allowed this measure, at quite a

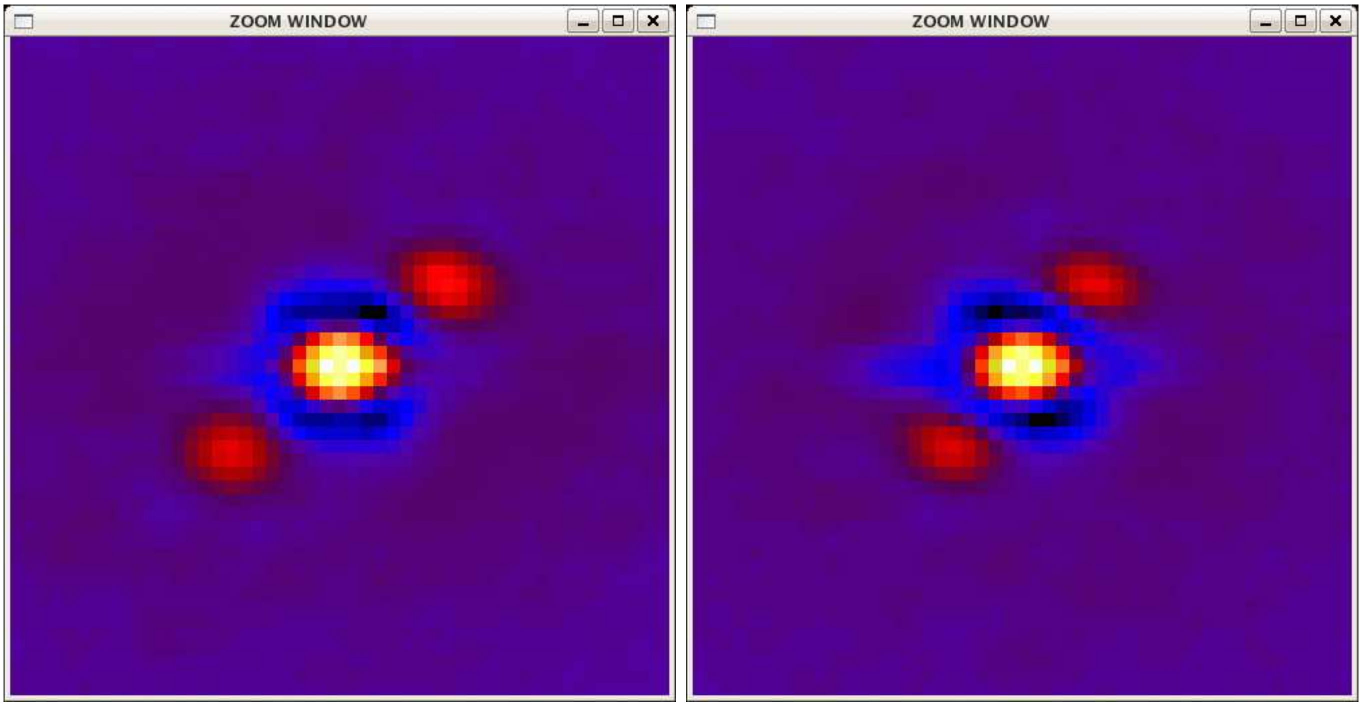

Fig. 4.— Panel a (left) is a directed vector autocorrelation of the subarray around the brighter (Aa,Ab) pair of Finsen 332 while Panel b (right) is the fainter (Ba,Bb) pairing. These were generated from the same 1982.7650 data shown in Figure 3c. The images, at the same scale, provide a vivid representation of the pairs' similar morphologies.

close separation, to be made; results are included in Table 1.

5. Discussion

5.1. Orbit Determination

The larger errors associated with both micrometry and eyepiece interferometry, as well as the small Δm and the geometric peculiarities of the systems as illustrated in Figure 2, make them quite difficult to distinguish. However, observations by speckle interferometry are characterized by much lower errors (see Hartkopf et al. 2001). Therefore, only the measures obtained with 2m or larger telescopes are utilized in the orbit analyses. The measures not included in the orbit determinations are indicated with notes in Table 1. The method of orbit calculation is the adaptive grid-search algorithm of Hartkopf et al. (1989), as modified by Mason et al. (1999). Briefly described, the Thiele-Innes elements (A,B, F and G) are calculated via an iterative three-dimensional grid-search of elements P, T_o, and e with the search parameter space decreasing as the elements converge. The remaining Campbell elements (a$''$, i, Ω, and ω) are then calculated directly. Observations are weighted using the scheme described in Hartkopf et al. (2001), which considers technique, observer expertise, the measured separation as a fraction of the telescope's Rayleigh limit, number of measures in a mean position, and any other notes the observer might have made with regard to quality. Table 4 gives the seven orbital elements along with their associated errors for both Aa,Ab and Ba,Bb. The degree of success realized by the ensemble following the rubrics of Hartkopf et al. (2001), summarized by the grade, is also given here. Table 5 gives predicted positions (ρ and θ) at half-year intervals for the next five years. These orbits are illustrated in Figures 5 (Aa,Ab) and 6 (Ba,Bb).

5.1.1. Radial Velocity Measures

One of the items of greatest interest to investigate was the initial mention of radial velocity variability; however, this did not prove helpful in setting limits on orbital parameters. The spectral types of the components (A or a little later for each of them) makes the measurement of radial velocity variability quite difficult due to the broad nature of the spectral features and the absence of many sharp metal lines. Plaskett et al. (1921) first noted variability and included STF2375 in their list of new spectroscopic binaries based on five observations obtained from June to October of 1920. Wilson (1953) added no new data in his catalog but gave it a quality rating of 'acceptable.' Palmer et al. (1968) added eight new velocities, but changed the mean by only 1 km s^{-1}. Evans (1979) in his revision of Wilson's catalog later gave it a quality rating of 'average.'

While most of the components in the multiple system are broad-lined A stars, the Bb component may be an F star with sharper lines and it is possible that near periastron it may exhibit variable radial velocity features.

5.1.2. Interpreting Unresolved Measures

As seen in Tables 1 and 2 and illustrated in Figure 2, there are two recent times in the short-period orbits when the pairs were predicted to be unresolved: Aa,Ab from 1964.6 to 1969 and again from 1991.7 to 1996.1, Ba,Bb from 1966.7 to 1972.1 and again from 2005.2 to 2010.7. The later two periods of predicted non-resolution corresponded to multiple null detections for both pairs, as indicated in Tables 1 and 2. These non-resolutions, while not utilized in determining these orbits, are completely consistent with the solutions.

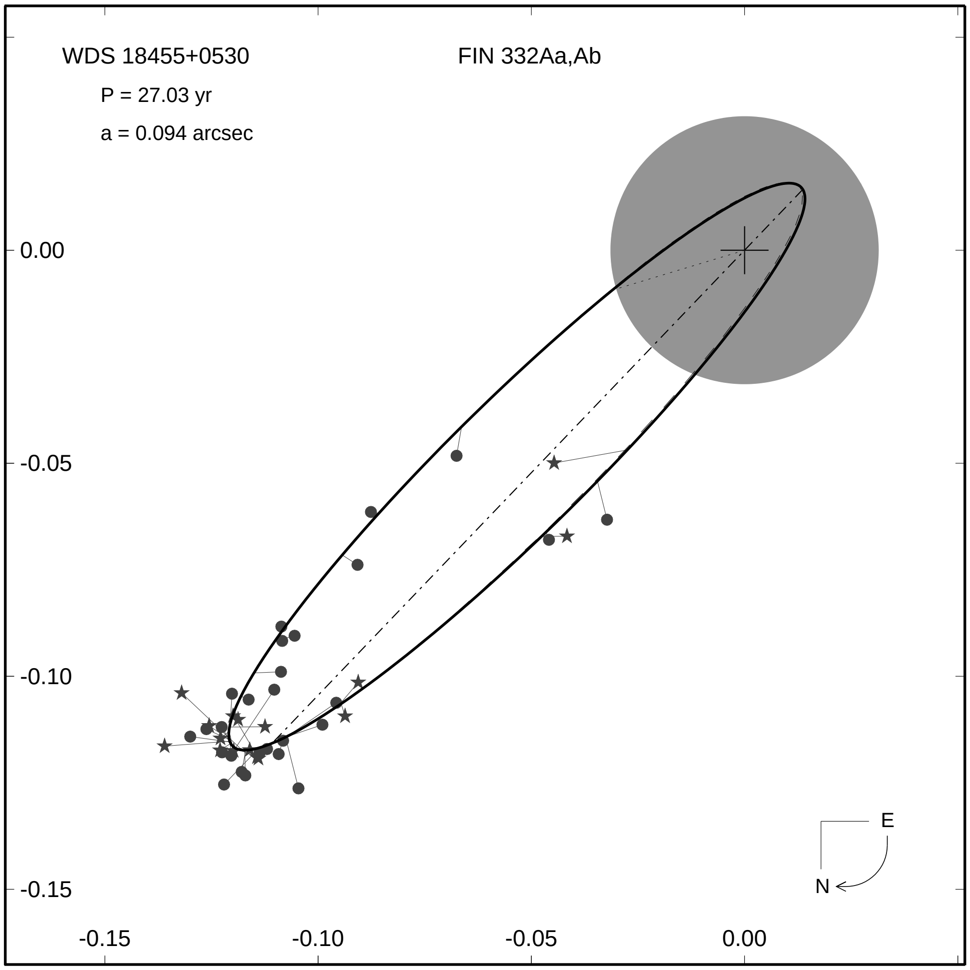

Fig. 5.— The relative orbit of FIN 332Aa,Ab, The different techniques are represented by filled circles for published measures and filled stars for new or newly corrected measures. Only data used in this orbit determination is plotted. The measures are connected to the predicted position by an O−C line. The dashed line through the origin is the line of nodes. The light grey circle is the Rayleigh resolution limit ($\frac{1.22\lambda}{D}$) of a 4m telescope. Unresolved measures from 4m class instruments are indicated by a dotted line drawn from the origin. The scale is in arcseconds and the direction of motion is indicated in the lower right corner. The barely distinguishable dashed curve is the short period solution of Mason & Hartkopf (2002).

5.2. Mutual Inclination

FIN 332 offers the rare possibility of determining the mutual inclination of orbits in a quadruple system whose subsystems are at the same hierarchical level. A first glance shows

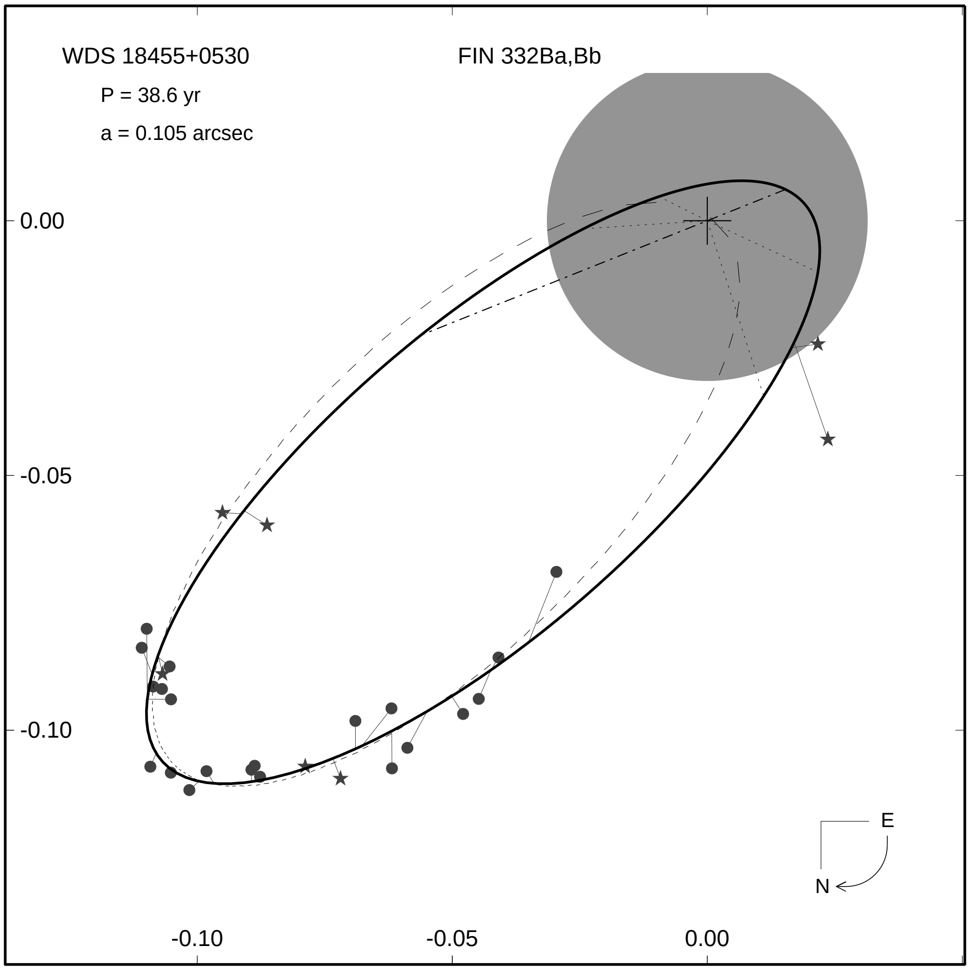

Fig. 6.— The relative orbit of FIN 332Ba,Bb. Symbols are the same as Figure 5. Note the larger number of unresolved measures, and the greater divergence from the Mason & Hartkopf (2002) short period solution.

that the individual orbital inclinations agree to with 1σ. However, the mutual inclination of their orbital planes is also dependent upon their nodal longitudes as given in the relation:

$$cos(\phi) = cos(i_{Aa,Ab})cos(i_{Ba,Bb}) + sin(i_{Aa,Ab})sin(i_{Ba,Bb})cos(\Omega_{Aa,Ab} - \Omega_{Ba,Bb}).$$

Inserting the values of (i,Ω)$_{Aa,Ab}$ and (i,Ω)$_{Ba,Bb}$ from Table 4 into this relation yields a mutual inclination of $\phi_{AB} = 25.2 \pm 12.2$ degrees. This indicates that the two orbits are more coplanar than not; however, if we adopt the threshhold for coplanarity defined by Fekel (1981) of $\phi < 15^{\circ}$ then these two orbits are within 1σ of being coplanar.

We have thus far assumed that the nodes specified by $\Omega_{Aa,Ab}$ and $\Omega_{Ba,Bb}$ are indeed the ascending nodes, but, regrettably, there is no spectroscopy to support that assumption. The two orbital inclinations reflect that both orbits are revolving in a retrograde sense, i.e. their position angles are decreasing with time. Interestingly, the wide and very long-period system is clearly moving in a direct sense with position angles increasing with time.

Because of the degeneracy of the Omegas, a second possible value for the mutual inclination of 49.3±19.6 degrees arises. While the eye is naturally attracted to the case of identical nodal quadrants, without radial velocity verification, which has already been shown to be a very challenging task, there remains the possibility that nature lacks the aesthetic of the eye. At this point, all we can state is that while the two orbital planes may be nearly coplanar, they are most certainly not nearly perpendicular.

5.3. Mass Sums

While both Aa and Ba are listed as spectral type A1V in the Multiple Star Catalog (Tokovinin 1997), the spectral types of the secondaries are not known. Given the small magnitude differences (discounting the Hipparcos Δm) it is conceivable that we have four A dwarf stars with expected mass sums of each pair between 5 and 6 $\mathcal{M}_{\odot}$. Unfortunately, given the large errors in the parallax and orbital elements these are of little help. The Aa,Ab solution gives a mass sum of $12{\pm}16\mathcal{M}_{\odot}$ while that of Ba,Bb is $7.6 \pm 9.0\mathcal{M}_{\odot}$. While their orbital elements can undoubtedly be improved, especially if they are resolved during periastron, the largest improvement may come from a more precise determination of their parallax.

5.4. Stitching 'Dum' and 'Dee' on their collars

For the first several decades since their discovery, the peculiar geometries of these systems made them nearly indistinguishable. If we compare their predicted position and subjectively qualify them as "similar" when their positions are approximately the same: $d\theta < 10°$ and $d\rho < 0\farcs05$ or both $< 0\farcs05$, i.e., unresolved, they would be qualified as "similar" for 33% of the next thousand years. While their appearances have diverged somewhat in recent years, by the middle of this century both pairs will again go through periastron within a few years of each other and FIN 332 Aa,Ab and Ba,Bb will again exemplify their Carrollian sobriquets.

We would like to acknowledge William S. Finsen and William van den Bos for their pioneering work in double star interferometry in the southern hemisphere. Also we would like to thank the USNO on-site archives for access to the voluminous correspondence between W.S. Finsen and C.E. Worley, head of the USNO visual double star program and Washington Double Star catalog projects for more than thirty years. The USNO speckle interferometry program has been supported by the National Aeronautics and Space Administration under reimburseable #NNH06AD70I, issued through the Terrestrial Planet Finder Foundation Science program. This research has made use of the SIMBAD database, operated at CDS, Strasbourg, France. Thanks are also extended to Ken Johnston and the U. S. Naval Observatory for their continued support of the Double Star Program.

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Table 1. Measurements of FIN 332Aa,Ab

Epoch	θ (∘)	ρ (″)	n	O−C (∘)	O−C (″)	Method	Reference	Notes
1953.73	316.5	0.153	5	3.0	−0.014	E	Finsen 1953	1
1953.74	315.7	0.15	1	2.2	−0.017	M	van den Bos 1956	1
1954.68	302.7	0.158	4	−10.2	−0.007	E	Finsen 1956	1
1955.72	309.8	0.144	3	−2.3	−0.017	E	Finsen 1956	1
1957.39	311.9	0.15	4	1.2	−0.002	M	van den Bos 1958b	1
1957.76	314.7	0.144	1	4.3	−0.005	E	Finsen 1959	1
1957.89	314.6	0.13	4	4.3	−0.018	M	van Biesbroeck 1960	1
1958.54	311.6	0.15	3	1.9	0.008	M	van den Bos 1960	1
1959.72	302.9	0.131	3	−5.6	0.000	E	Finsen 1960	1
1960.564	318.2	0.14	6	10.8	0.019	M	van Biesbroeck 1965	1
1960.72	298.9	0.137	1	−8.3	0.018	E	Finsen 1961	1
1961.57	312.6	0.11	5	6.7	0.003	M	van den Bos 1962	1
1961.73	297.8	0.112	3	−7.8	0.008	E	Finsen 1962	1
1962.51	314.7	0.10	4	10.7	0.008	M	van den Bos 1963a	1
1962.72	309.2	0.114	5	5.7	0.026	E	Finsen 1963	1
1963.38	unresolved		1	(301.5)	(0.076)	M	van den Bos 1963b	1,2
1963.728	313.0	0.106	4	12.8	0.037	E	Finsen 1964a	1
1964.726	unresolved		1	(294.1)	(0.046)	E	Finsen 1965	1,2
1966.758	unresolved		1	(157.0)	(0.016)	E	Finsen 1967	1,2
1968.791	unresolved		1	(334.2)	(0.040)	E	Finsen 1969	1,2
1971.531	307.0	0.15	1	−15.2	0.046	M	Walker 1972	1
1975.48	316.9	0.12	3	−0.6	−0.031	M	Heintz 1978	1
1976.2992	318.1	0.143	1	1.2	−0.014	Sp	McAlister 1978	
1976.3702	318.5	0.149	1	1.7	−0.008	Sp	McAlister & Hendry 1982a	
1976.3728	320.5	0.164	1	3.7	0.007	Sp	McAlister & Hendry 1982a	
1976.4549	317.4	0.161	1	0.6	0.004	Sp	McAlister 1978	
1977.3340	316.9	0.158	1	0.8	−0.004	Sp	McAlister & Hendry 1982a	
1977.4815	316.4	0.162	1	0.4	−0.000	Sp	McAlister 1979	
1977.4871	316.2	0.164	1	0.2	0.002	Sp	McAlister 1979	
1977.521	312.9	0.18	3	−2.9	−0.017	M	Walker 1985	1
1977.6400	315.9	0.175	1	0.0	0.012	Sp	McAlister & Hendry 1982a	
1978.5410	316.2	0.170	1	1.0	0.004	Sp	McAlister & Fekel 1980	
1978.6147	316.6	0.170	1	1.4	0.004	Sp	McAlister & Fekel 1980	
1979.3601	314.0	0.170	1	−0.6	0.003	Sp	McAlister & Hendry 1982b	
1979.5321	313.2	0.151	1	−1.3	−0.016	Sp	McAlister & Hendry 1982b	
1979.7725	312.5	0.166	1	−1.8	−0.001	Sp	McAlister & Hendry 1982b	

Table 1—Continued

Epoch	θ ($\circ$)	ρ ($''$)	n	O−C ($\circ$)	O−C ($''$)	Method	Reference	Notes
1980.4769	311.4	0.173	1	−2.4	0.006	Sp	McAlister et al. 1983	
1980.4794	314.7	0.169	1	0.9	0.002	Sp	McAlister & Hartkopf 1984	
1980.7173	311.0	0.159	1	−2.7	−0.008	Sp	McAlister et al. 1983	
1980.7199	311.8	0.169	1	−1.9	0.002	Sp	McAlister et al. 1983	
1981.356	313.2	0.186	1	0.1	0.020	P	Tokovinin 1982	1
1982.5029	316.4	0.165	1	4.0	0.003	Sc	This paper	3
1982.5248	315.2	0.160	1	3.0	−0.002	Sc	Fu et al. 1997	1
1982.7650	313.0	0.162	2	0.8	0.001	Sc	This paper	3
1983.4203	312.3	0.157	1	0.7	0.001	Sc	McAlister et al. 1987a	4
1984.3760	312.4	0.147	1	1.5	−0.005	Sc	Hartkopf et al. 2000	5
1984.783	335.9	0.127	1	25.5	−0.022	P	Tokovinin & Ismailov 1988	1
1985.4816	310.7	0.139	1	0.8	−0.004	Sc	McAlister et al. 1987a	4
1985.5231	310.3	0.142	1	0.5	−0.001	Sc	McAlister et al. 1987b	4
1985.7440	318.3	0.137	1	8.8	−0.004	P	Tokovinin & Ismailov 1988	1
1985.8424	309.2	0.140	1	−0.3	0.000	Sc	McAlister et al. 1987a	4
1987.7618	309.2	0.117	1	2.0	−0.001	Sc	McAlister et al. 1989	4
1988.6655	305.1	0.107	1	−0.7	0.001	Sc	McAlister et al. 1990	
1990.2734	305.6	0.083	1	3.6	0.005	Sc	Hartkopf et al. 1992	
1991.2500	unresolved			(297.8)	(0.058)	H	ESA 1997	2,6
1992.3105		<0.038	1	(286.8)	(0.032)	Sc	This Paper	2,7
1996.3214	318.2	0.067	1	−11.4	0.013	Sc	This Paper	8
1996.6930	333.0	0.071	1	5.5	0.007	Sc	Hartkopf et al. 2000	
1997.3945	326.0	0.082	1	1.1	0.001	S	Balega et al. 1999	
1997.4630	328.2	0.079	1	3.5	−0.003	Sc	This Paper	9
2001.4988	319.4	0.144	1	1.0	0.002	S$\star$	This Paper	10
2001.5697	318.2	0.136	1	−0.1	−0.007	S$\star$	This Paper	11
2005.8652	308.2	0.168	1	−6.7	0.002	S$\star$	This Paper	10
2006.2001	315.3	0.165	1	0.7	−0.002	S$\star$	This Paper	11
2006.5640	316.1	0.165	1	1.8	−0.002	S$\star$	This Paper	9
2007.3174	313.6	0.170	1	−0.2	0.003	S$\star$	This Paper	9
2007.5879	310.5	0.179	2	−3.1	0.012	S$\star$	This Paper	10
2007.8010	311.6	0.168	1	−1.9	0.001	S$\star$	This Paper	9
2008.4529	313.0	0.168	4	−0.1	0.003	S$\star$	This Paper	10
2008.5371	314.4	0.168	2	1.5	0.003	S	Tokovinin et al. 2010	12
2008.7721	314.8	0.159	1	2.0	−0.006	S	Tokovinin et al. 2010	13
2008.8712	316.9	0.192	2	4.2	0.028	S$\star$	This Paper	1, 14

Table 1—Continued

Epoch	θ ($\circ$)	ρ ($''$)	n	O−C ($\circ$)	O−C ($''$)	Method	Reference	Notes
2009.2607	312.3	0.162	2	−0.1	−0.001	S	Tokovinin et al. 2010	12

Methods: E = eyepiece interferometer, H = Hipparcos observation, M = micrometer, P = phase grating interferometer, S = speckle interferometer, Sp = photographic speckle camera of McAlister (1977), Sc = ICCD speckle camera of McAlister et al. (1987a), S$\star$ = USNO speckle camera of Mason et al. (2009).

1 : Measure not used in new orbit solution.

2 : Here Columns 4 & 5 give the predicted position of the secondary relative to the primary.

3 : Measure obtained by re-reduction of CCD subarray. See §4.4.

4 : The original calibration was corrected in McAlister et al. (1989) and this corrected measure first published in McAlister & Hartkopf (1988).

5 : Re-reduction of data yielded improved SNR and allowed this measure to be made.

6 : No measure of this subsystem was published in the Hipparcos Catalogue.

7 : The other pair, Ba,Bb (see Table 2) was measured at this time, so this is judged to be a reliable null detection.

8 : Measure inadvertently left out of Hartkopf et al. (2000).

9 : Observation made on Mt. Wilson 100$''$.

10 : Observation made on KPNO 4m.

11 : Observation made on CTIO 4m.

12 : Δm is 0.9$\pm$0.4 in Strömgren y.

13 : Δm is 1.3 in Hα.

14 : Observation made on NOFS 61$''$.

Table 2. Measurements of FIN 332Ba,Bb

Epoch	θ (∘)	ρ (″)	n	O−C (∘)	O−C (″)	Method	Reference	Notes
1953.73	315.3	0.148	5	1.3	−0.002	E	Finsen 1953	1
1953.74	315.1	0.14	1	1.1	−0.010	M	van den Bos 1956	1
1954.68	317.6	0.144	4	4.6	−0.005	E	Finsen 1956	1
1955.72	314.9	0.141	3	3.0	−0.007	E	Finsen 1956	1
1957.39	310.0	0.16	4	−0.1	0.017	M	van den Bos 1958	1
1957.73	300.	0.15	2	−9.7	0.008	M	Muller 1958	1
1957.73	306.	0.14	1	−3.7	−0.002	M	Muller 1958	1
1957.76	308.3	0.147	1	−1.4	0.005	E	Finsen 1959	1
1957.89	313.8	0.12	4	4.3	−0.021	M	van Biesbroeck 1960	1
1958.54	312.3	0.15	3	3.6	0.011	M	van den Bos 1960	1
1959.72	312.3	0.124	3	5.1	−0.009	E	Fin1960b	1
1960.564	312.4	0.13	6	5.8	0.003	M	van Biesbroeck 1965	1
1960.72	310.9	0.139	1	5.1	0.013	E	Finsen 1961	1
1961.57	311.2	0.13	5	6.7	0.010	M	van den Bos 1962	1
1961.73	311.0	0.126	3	6.7	0.007	E	Finsen 1962	1
1962.51	312.0	0.11	4	9.1	−0.002	M	van den Bos 1963a	1
1962.72	320.8	0.123	5	18.3	0.014	E	Finsen 1963	1
1963.38	unresolved		1	(301.1)	(0.102)	M	van den Bos 1963b	1,2
1963.728	323.6	0.113	4	23.3	0.015	E	Finsen 1964a	1
1964.726	unresolved		1	(297.5)	(0.084)	E	Finsen 1965	1,2
1966.436	276.7	0.26	1	−11.7	0.210	M	Walker 1969	1
1966.758	unresolved		1	(284.8)	(0.041)	E	Finsen 1967	1,2
1968.791	unresolved		1	(72.1)	(0.023)	E	Finsen 1969	1,2
1971.504	90.0	0.15	1	84.9	0.105	M	Walker 1972	1
1976.4549	336.9	0.075	1	−0.3	−0.015	Sp	McAlister 1978	
1977.4815	334.6	0.095	1	0.2	−0.003	Sp	McAlister 1979	
1977.4870	334.6	0.104	1	0.2	0.006	Sp	McAlister 1979	
1977.521	317.0	0.12	3	−17.2	0.022	M	Walker 1985	1
1978.6147	333.8	0.108	1	1.9	0.002	Sp	McAlister & Fekel 1980	
1979.3601	330.5	0.119	1	0.1	0.008	Sp	McAlister & Hendry 1982b	
1980.4769	330.2	0.124	1	1.8	0.006	Sp	McAlister et al. 1983	
1981.356	321.5	0.111	1	−5.4	−0.012	P	Tokovinin 1982	1
1981.4681	327.2	0.114	1	0.4	−0.009	Sp	McAlister et al. 1984	
1981.6975	325.0	0.120	1	−1.5	−0.005	Sp	McAlister et al. 1984	
1982.5029	326.8	0.131	1	1.5	0.002	Sc	This Paper	3
1982.7650	323.7	0.133	1	−1.2	0.003	Sc	This Paper	3

Table 2—Continued

Epoch	θ ($\circ$)	ρ ($''$)	n	O−C ($\circ$)	O−C ($''$)	Method	Reference	Notes
1984.783	329.4	0.103	1	7.2	−0.035	P	Tokovinin & Ismailov 1988	1,4
1985.4816	321.3	0.140	1	−0.1	−0.001	Sc	McAlister et al. 1987a	5
1985.5231	320.4	0.139	1	−1.0	−0.002	Sc	McAlister et al. 1987b	5
1985.7440	308.3	0.100	1	−12.8	−0.042	P	Tokovinin & Ismailov 1988	1,4
1985.8424	320.4	0.140	1	−0.6	−0.002	Sc	McAlister et al. 1987a	5
1987.7618	317.8	0.146	1	−1.1	−0.001	Sc	McAlister et al. 1989	
1988.6655	317.8	0.151	1	−0.1	0.003	Sc	McAlister et al. 1990	
1990.2734	315.9	0.151	1	−0.3	0.001	Sc	Hartkopf et al. 1992	
1991.2500	308.	0.16	0	−7.1	−0.009	H	ESA 1997	1,6
1992.3105	314.5	0.153	1	0.4	0.003	Sc	Hartkopf et al. 1994	
1995.6008	306.1	0.136	1	−4.5	−0.009	Sc	Hartkopf et al. 1997	7
1995.6061	311.8	0.141	1	1.2	−0.004	Sc	Hartkopf et al. 2000	
1996.3215	310.7	0.141	1	1.0	−0.001	Sc	Hartkopf et al. 2000	
1996.3270	310.1	0.142	1	0.4	−0.000	Sc	Hartkopf et al. 2000	
1996.7012	307.1	0.139	1	−2.2	−0.002	Sc	Hartkopf et al. 2000	
1997.3945	309.7	0.137	1	1.2	−0.001	S	Balega et al. 1999	
1997.4630	309.8	0.139	1	1.4	0.002	Sc	This Paper	8
2001.4988	301.1	0.111	3	−1.1	0.003	S$\star$	This Paper	9
2001.5697	304.7	0.105	1	2.6	−0.002	S$\star$	This Paper	10
2005.8652		<0.038	1	(273.7)	(0.024)	S$\star$	This Paper	2,9,11
2006.2001		<0.038	1	(243.3)	(0.011)	S$\star$	This Paper	2,10,11
2006.5640		<0.060	1	(127.2)	(0.013)	S$\star$	This Paper	2,11,12
2007.3174		<0.060	1	(76.7)	(0.023)	S$\star$	This Paper	2,11,12
2007.5879		<0.038	1	(65.0)	(0.024)	S$\star$	This Paper	2,9,11
2007.8010		<0.060	1	(56.7)	(0.025)	S$\star$	This Paper	2,11,12
2008.4615	28.8	0.049	2	−6.7	0.020	S$\star$	This Paper	9
2008.5371	41.8	0.033	1	8.4	0.003	S	Tokovinin et al. 2010	13
2008.8658		<0.098	1	(25.6)	(0.033)	S$\star$	This Paper	2,14
2009.2607		<0.050	1	(17.8)	(0.036)	S	Tokovinin et al. 2010	2,15

Methods: E = eyepiece interferometer, H = Hipparcos observation, M = micrometer, P = phase grating interferometer, S = speckle interferometer, Sp = photographic speckle camera of McAlister (1977), Sc = ICCD speckle camera of McAlister et al. (1987a), S$\star$ = USNO speckle camera of Mason et al. (2009).

1 : Measure not used in new orbit solution.

2 : Here Columns 4 & 5 give the predicted position of the secondary relative to the primary.

3 : Measure obtained by re-reduction of CCD subarray. See §4.4.

4 : Published position angle was 59°.4, and 38°.3 and given zero weight in orbit determination. See §4.1.

5 : The original calibration was corrected in McAlister et al. (1989) and this corrected measure first published in McAlister & Hartkopf (1988).

6 : The H_p magnitude difference is 0.76±0.15.

7 : Assigned in error to Aa,Ab in Hartkopf et al. (1997).

8 : Observation made on Mt. Wilson 100″.

9 : Observation made on KPNO 4m.

10 : Observation made on CTIO 4m.

11 : The other pair, Aa,Ab (see Table 1) was measured at this time, so this is judged to be a reliable null detection.

12 : Observation made on Mt. Wilson 100″. Not plotted in Figure 6.

13 : Δm is 0.5 in Strömgren y.

14 : Observation made on NOFS 61″. Not plotted in Figure 6.

15 : Observation obtained on the SOAR 4.2m telescope. While Ba,Bb was previously resolved when it was closer according to A. Tokovinin: "Bab could be partially resolvable, but in the AD [Atmosheric Dispersion] direction. Fits do not converge, so it remains unresolved. The AD was 3.2 pixels, so if the pair was under 50mas or so, the negative result could be explained."

Table 3. Measurements of STF2375AB

Epoch	θ ($\circ$)	ρ ($''$)	n	Method	Notes
1997.4657	119.6	2.590	1	Sc	1
2004.2019	122.9	2.496	1	S$\star$	2
2006.1974	120.1	2.549	1	S$\star$	3
2006.5640	119.7	2.512	1	S$\star$	1
2007.3174	119.6	2.484	1	S$\star$	1
2007.5879	118.2	2.537	1	S$\star$	4
2007.8010	118.2	2.537	1	S$\star$	1
2008.4569	119.5	2.504	3	S$\star$	4
2008.8549	119.1	2.618	3	S$\star$	2
2008.8712	119.1	2.569	3	S$\star$	2

Methods: Sc = ICCD speckle camera of McAlister et al. (1987a), S$\star$ = USNO speckle camera of Mason et al. (2009)

4 : Observation made on KPNO 4m.

3 : Observation made on CTIO 4m.

1 : Observation made on Mt. Wilson 100$''$.

2 : Observation made on NOFS 61$''$.

Table 4. Orbital Elements of FIN 332Aa,Ab & Ba,Bb

Element	FIN 332Aa,Ab	FIN 332Ba,Bb
Period; P (yrs)	27.03 $\pm$ 0.67	38.6 $\pm$ 1.2
Semi-major axis; a$''$	0.094$\pm$ 0.019	0.105$\pm$ 0.015
Inclination; i ($^\circ$)	106. $\pm$20.	117.2 $\pm$ 9.5
Longitude of Node; Ω ($^\circ$)	136.2 $\pm$ 4.2	111.8 $\pm$ 5.7
Epoch of Periastron; T_o (yrs)	1994.20 $\pm$ 0.98	1967.9 $\pm$ 1.9
Eccentricity; e	0.79 $\pm$ 0.34	0.867$\pm$ 0.034
Longitude of Periastron; ω ($^\circ$)	10. $\pm$16.	311.2 $\pm$ 8.3
Grade	3	3

Table 5. Ephemerides of FIN 332Aa,Ab & Ba,Bb

Epoch	FIN 332Aa,Ab		FIN 332Ba,Bb	
	θ (deg)	ρ (arcsec)	θ (deg)	ρ (arcsec)
2010.0	312.0	0.160	6.8	0.043
2010.5	311.6	0.158	1.2	0.048
2011.0	311.2	0.155	356.6	0.053
2011.5	310.8	0.151	352.8	0.058
2012.0	310.3	0.147	349.6	0.063
2012.5	309.9	0.143	346.9	0.068
2013.0	309.4	0.139	344.5	0.072
2013.5	308.8	0.134	342.4	0.077
2014.0	308.3	0.128	340.5	0.081
2014.5	307.6	0.122	338.8	0.085